Curved Track Centerline and Obstacle Clearance Calculation Methodology

NMRA Technical Note TN-7

By: Van S. Fehr

July 2017

ACKNOWLEGEMENTS

I wish to thank Didrik A. Voss, MMR, Manager, S&C Dept. for his guidance and significant contributions to this effort. His insightful suggestion to use Dimension **A** from **NMRA RP-7.1** as the obstacle contact half-width was pivotal in finalizing the logic underlying the methodology developed in this Technical Note. That insight led to the selection of one-half the tangent track centerline spacing as the equipment contact half-width, in turn ensuring proper clearance between equipment operating on concentric curved tracks.

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July 2017

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Introduction

This NMRA Technical Note (TN) develops a methodology for calculating the centerline spacing of concentric circular tracks. Further, the methodology calculates the obstacle clearance for trackside objects, measured from curved track centerlines.

At the time of this writing, NMRA Standard S-6 [1][1], S-7 [2] and S-8 [3] are under review, with the expectation of their replacement by a new Recommended Practices series introduced by NMRA RP-7. This TN supports that review with an engineering analysis intended to form the basis for the revised content presented in the new RP-7. Two companion computer applications (*apps*, in today's computing jargon) implement the equations developed in this TN.

The first app is a comprehensive MS-Excel spreadsheet **NMRA *Track Center and Obstacle Clearance Calculator* DRAFT 04.xlsm**. It is intended primarily as a tool to help the NMRA Standards and Conformance Department establish the underlying engineering and prepare content the new NMRA RP-7 series.

The second is an HTML application **NMRA *Track Center and Obstacle Clearance Calculator* DRAFT 04.html.** It is intended to help NMRA members easily make their own spacing calculations, if desired. It is designed to operate on any digital device that can access the NMRA website. These apps are subject to change, and would not be finalized until the new RP-7 series is ready for posting on the NMRA website.

Background

Prototype railroads specify track centerline and adjacent obstacle clearance in accordance with practices established by their own in-house practices, and by railroad associations (e.g., AREA/AREMA). Regardless of their own practices, railroads first comply with federal or state laws that often require minimum spacing values. The spacing values take varying equipment widths into account with a reasonably acceptable margin to ensure contact does not occur.

Like the prototype, the NMRA recommends obstacle clearance in the form of clearance diagrams it presents in the new RP-7.1. The obstacle clearance for tangent (straight) tracks is the half-width of the clearance diagram, which RP-7.1 labels as dimension **A**. Similarly, the NMRA recommends tangent track centerline spacing in RP-7.2, along with curved track centers obtained using the equations developed in this TN. These spacing values (prototype and NMRA) are sufficient to accommodate all equipment and maintain acceptable clearance (actual space).

Figure 1 (not to scale) diagrams rolling equipment on parallel tangent tracks. In this, and other figures that follow, green lines represent the equipment, red lines trackside obstacles, and blue lines track centerlines. All equipment moves safely when construction meets both obstacle clearance and tangent track spacing requirements. Equipment length and height vary considerably, but do not affect safe operation on parallel tangent tracks.

Figure 2 (not to scale) diagrams rolling equipment on concentric curved tracks. Track centerline radii are drawn excessively small to clarify the geometric relationships. On curved track, the equipment end corners swing outward and the inner side swings inward from the centerline. Both effects require increased obstacle clearance on curved tracks to ensure acceptable clearance. For the same reason, both effects require increased curved track centerline spacing.

In many early 20th century AREA documents, the AREA makes statements like this one, referring to bridge and tunnel clearance diagram half-widths:

"On curved track these widths are increased so as to provide the same clearance for rolling stock 80 ft. in length, 14 feet high, and 60 ft. center to center of tracks."

Tracks is no doubt a typographical error and should be *trucks*, or *bolsters*. In the second quarter of the 20th century the AREA modified the length of this rolling stock to 85 ft., but retained the other dimensions. Another statement, referring the tangent track centerline spacing, is:

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[1] Numbers in square brackets indicate like-numbered resources listed in the **References** section of this TN.

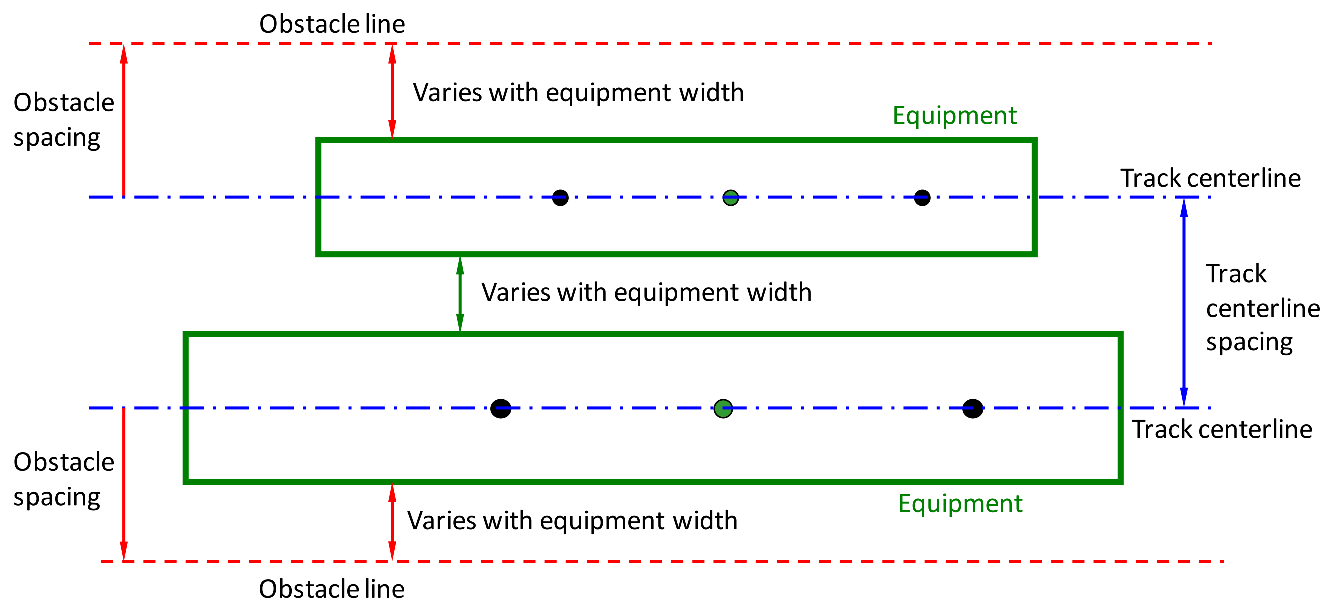

Figure 1: Equipment on Tangent Tracks (top-view)

"... while corrections should be made in all cases on curves for overhang and super-elevation."

The rolling stock dimensions in the first quote are likely for the longest passenger car in service at the time. Equipment (like passenger cars) having the largest truck pivot distance will require the largest obstacle clearance increase on the inside of a curve. In those days, the AREA made no mention of steam locomotives, whose length from the outer driver axles to the long end (discussed in the next section) can swing considerably outward on the outside of a curve. Prototype mainline curves are normally under ten degrees[2], and rarely exceed twenty degrees in other locations. Because of such low curvature, the AREA (and perhaps others) may have decided that passenger car dimensions were adequate for spacing increase determination.

Model railroad curves, however, are typically much sharper than the prototype, and can exceed 45 degrees of curvature. Thus, they require substantially larger spacing increases. That also means that the out-swing of steam engines, and any other equipment with long ends, must not be ignored. The equations developed below calculate track spacing and obstacle clearance for curved track, given the radius, the pertinent dimensions of ***two*** different kinds of equipment, the specified tangent track spacing, and the specified obstacle clearance.

The first AREA statement (… "*same clearance…*") means the tangent track centerline spacing and the tangent track obstacle clearance on curves must both increase *beyond* the tangent track values to accommodate the equipment out- and in-swing on curves. The second statement (… "*corrections… for… superelevation*") means the inward tilt of equipment must also be included in that increase. Taken together, this means that any equations that calculate curved track spacing values must, in the limit of zero curvature (infinite radius) and zero superelevation (at zero curvature), reduce to the tangent track spacing values.

Equipment Envelope Geometry

Freight cars, passenger cars and diesel locomotives normally have two trucks whose bolster pivots lie on the equipment's longitudinal centerline. The track centerline, curved or tangent, passes directly under the two pivot points[3], orienting the equipment body structure relative to the track.

Viewed from the top, all equipment, from a velocipede to an articulated steam locomotive, is nominally box-shaped. Using known dimensions, it is possible to define a three-dimensional rectangular box *equipment*

[2] Track curvature is the angle subtended by a 100-foot centerline chord, expressed as "degree-of-curvature."

[3] This is not strictly true on a curve, but for purposes here, the error is negligible.

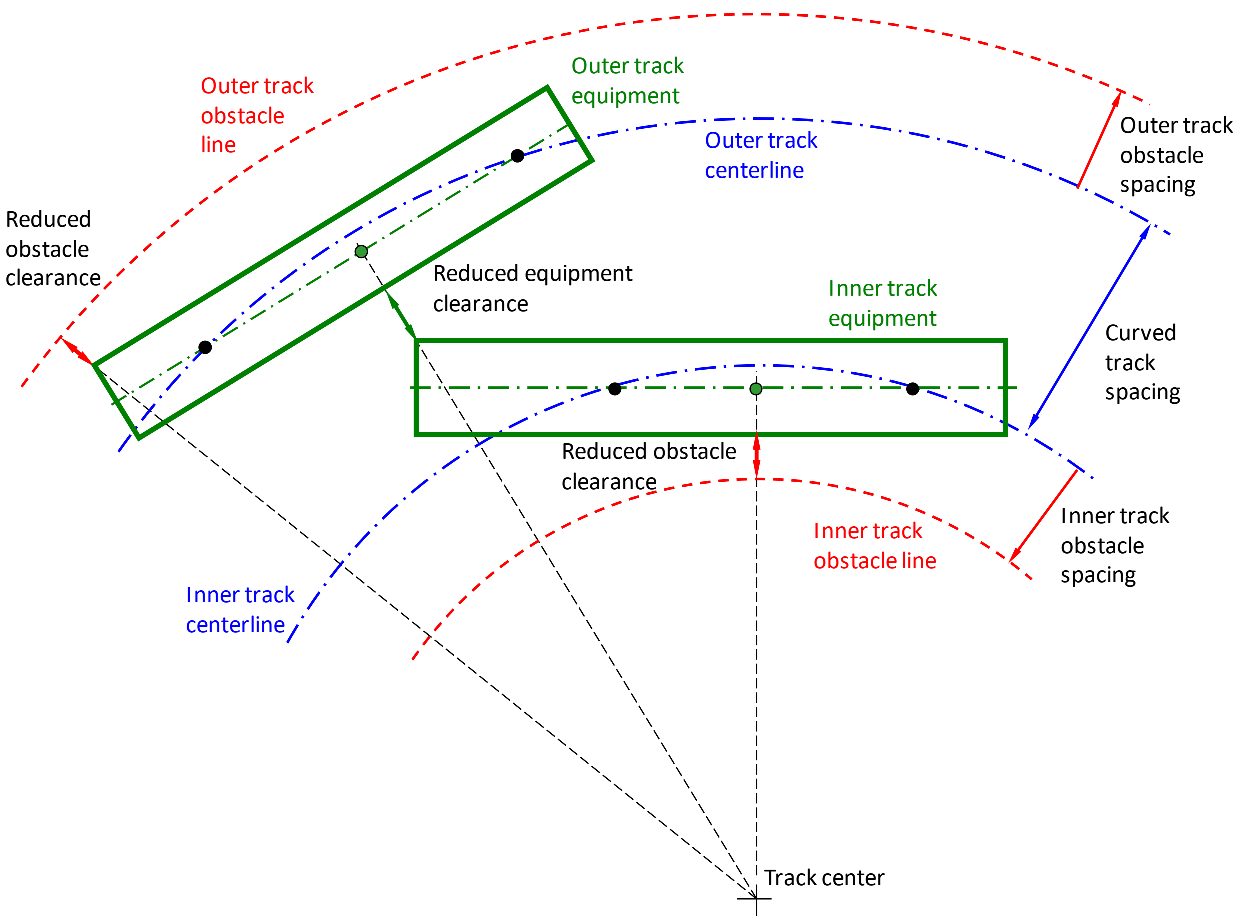

Figure 2: Equipment on Curved Tracks (top-view)

envelope that fully encompasses equipment geometry with adjustments to include any localized features. Figure 3 illustrates the equipment envelope for any kind of equipment. Hay [4] uses the top view of this envelope in his limited discussions of the effects of operating on a curve. Table 1 in the **APPENDIX** defines the mathematical variables used in Figure 3 and elsewhere in this TN.

For analysis, any equipment geometry is thus defined by an equipment envelope that has two pivots separated by a known distance. The distances from the pivot points to the nearest end of the equipment envelope are also known. These end distances are most often equal, but may be substantially different on steam locomotives, less so on some diesels, and perhaps not present on some modern articulated passenger or freight cars. Figure 1 labels them **Long End** and **Short End**. On a curve, the corner of the long end of the equipment extends further outward than the short end, so only the long end influences spacing.

Steam engines are a bit different, as Figure 4 illustrates. A typical steam locomotive has a set of drivers supported in a frame assembly that is rigidly mounted (non-pivoting) to the body structure. On a curve, the track centerline passes directly under the axle mid-points of the outermost drivers. The inner drivers have some axial play or are sometimes flangeless to accommodate track curvature, which may also employ gauge-widening for this purpose. Therefore, the wheelbase of the outermost driver axles defines the location of the two equivalent "pivots," and thus orient the locomotive body on the curved track. Leading and/or trailing trucks, when present, simply align with the track centerline and do not affect the locomotive body orientation.

On a typical *articulated* steam locomotive, one engine driver frame, normally the forward engine, hinges relative to the locomotive body structure, while the other engine frame, usually the rear, remains fixed to it. Thus, the hinged (articulated) engine frame notionally behaves like a leading or trailing truck and does not alter the orientation of the body structure on the track. Operating on a curve, the outward overhang of the front

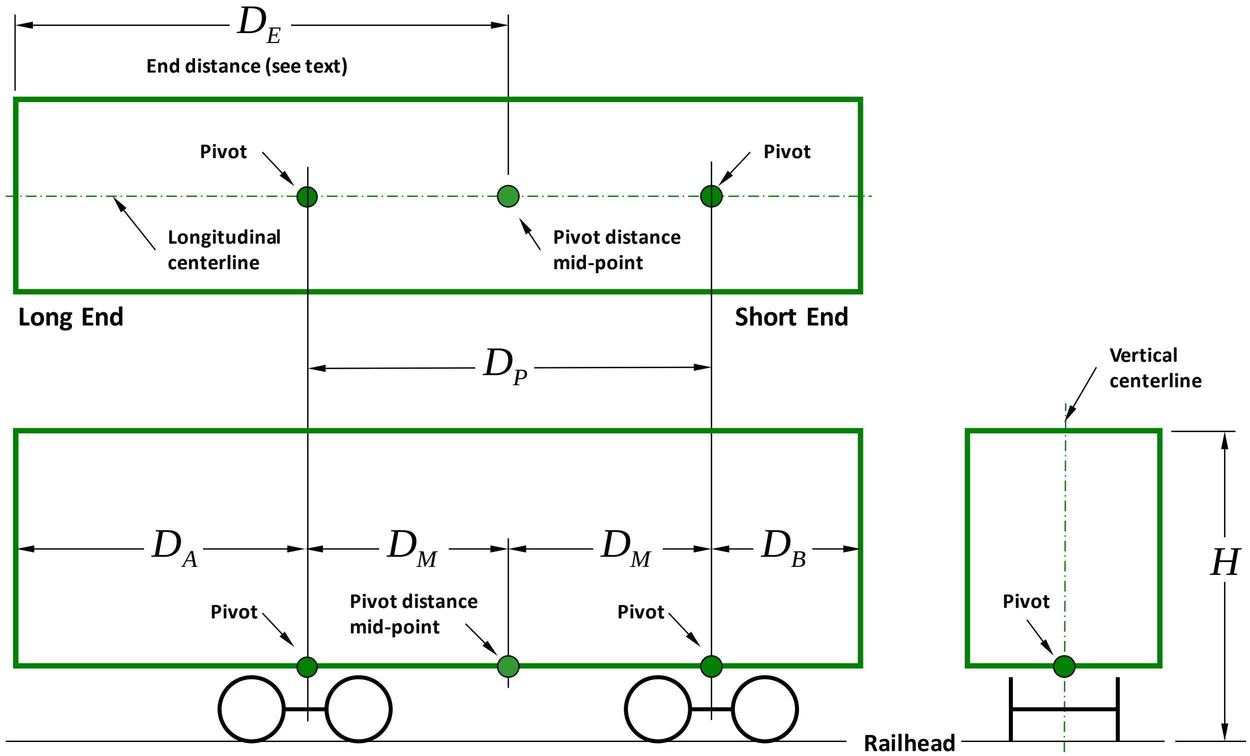

Figure 3: Equipment Envelope (three-view)

of an articulated steam locomotive is usually greater than that of a non-articulated steam locomotive. Some articulated locomotive models, mostly those made of brass, follow prototype construction. Other models, those made mostly from plastic, often mount each engine on its own pivoting bolster allowing operation on sharper, non-prototypical, curves.

The pivot distance is also important because it forms a chord on a curved track centerline. On a curve, the radius to the inner side of the equipment envelope is on a line perpendicular to this chord and passing though the pivot mid-point distance. On a curve, the longer the pivot distance, the further the equipment side moves inward from the track centerline.

The height of the equipment above the railhead is also important. On a curve with superelevation, the top of the equipment tilts inward towards the curve center. This requires increased clearance on the inner side of the curve and must be considered when determining curved track and obstacle clearance.

Using this box-shaped equipment envelope is conservative because the widest and highest parts of the equipment are not necessarily at the corners or on the corner edges.

Analysis Preliminaries

Two concepts need explanation before proceeding with derivations of the spacing equations. The first is the effect of superelevation, and the second the idea of contact half-widths.

Superelevation

Track centers and obstacle clearance on curved track depend not only on equipment dimensions, but on the presence, or absence, of superelevation. When no superelevation is present, the entire inner side of the equipment remains vertical and displaces inward away from the curved track centerline, as discussed earlier.

In the prototype, superelevation depends on train speed and track curvature, and decreases to zero when curvature decreases to zero. Unlike the prototype, model railroads never require superelevation, but modelers using it do so for appearance, and typically use a constant value regardless of curvature. For that reason, the superelevation height is treated as a constant in the equations that follow.

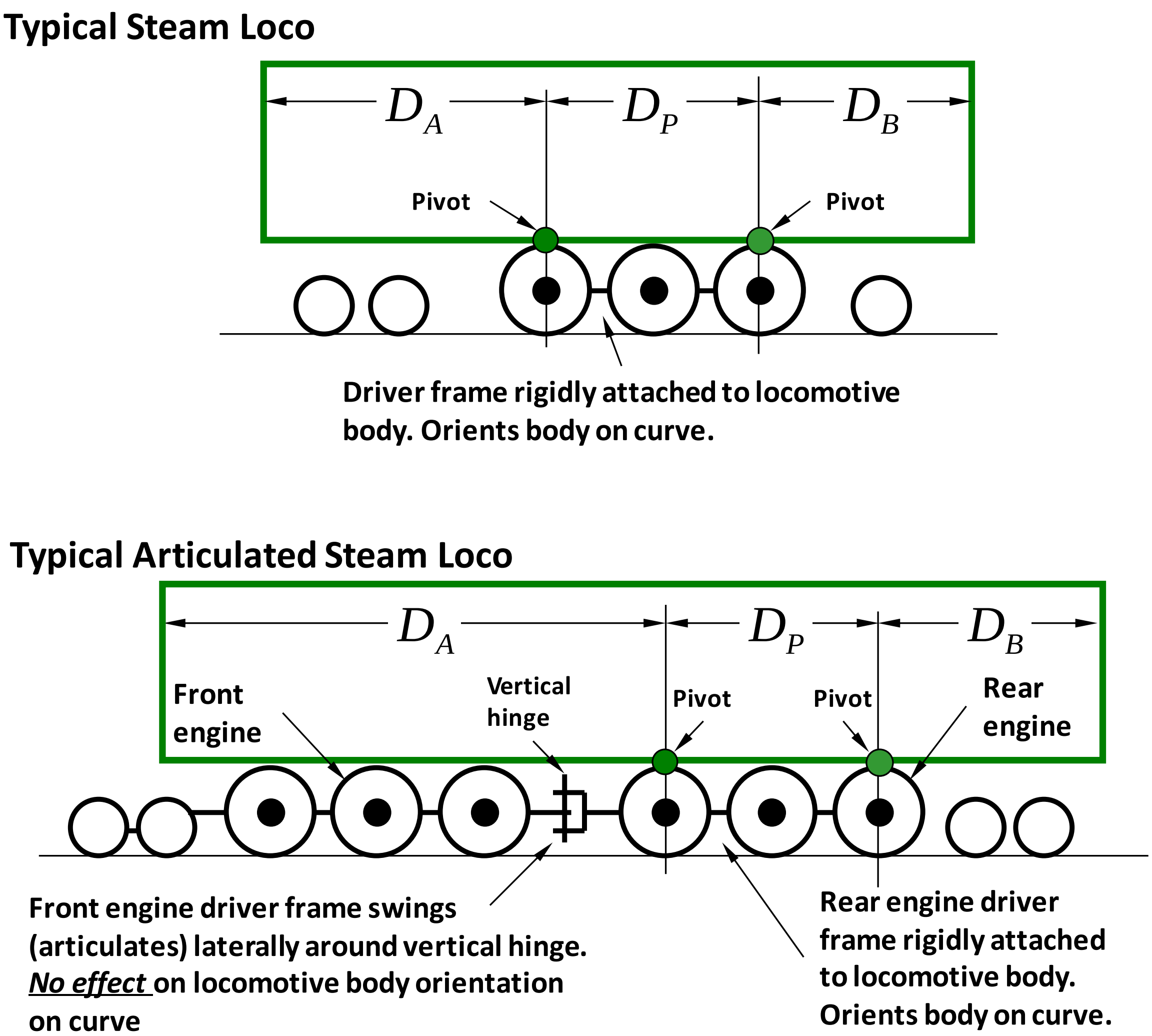

Figure 4: Steam Locomotive Pivot Locations

When superelevation is present, Figure 5 shows that the equipment envelope (viewed from the end) also tilts inward. The equipment tilts through an angle[4] defined by the superelevation height and the track gauge. Then, by similar triangles:

$$\frac{d}{H} = \frac{e}{G} \qquad (1)$$

Solve equation (6) for the distance the upper inside corner tilts towards the curve center, d :

$$d = \frac{e}{G} H \qquad (2)$$

In Figure 5, notice that the lower outside corner of the equipment envelope, being much closer to the axis of rotation, tilts inward a much smaller amount, δ , compared to the upper inside corner tilt, d . Because δ is so small, and the corner moves away from anything on the outside, δ can be safely ignored.

[4] This is an approximation because the equipment tilts around an axis parallel to a line through the pivots, not around an axis tangent to the track radius. Unless the track radius is near one-half of the pivot distance, which is highly unlikely, the approximation error is negligible.

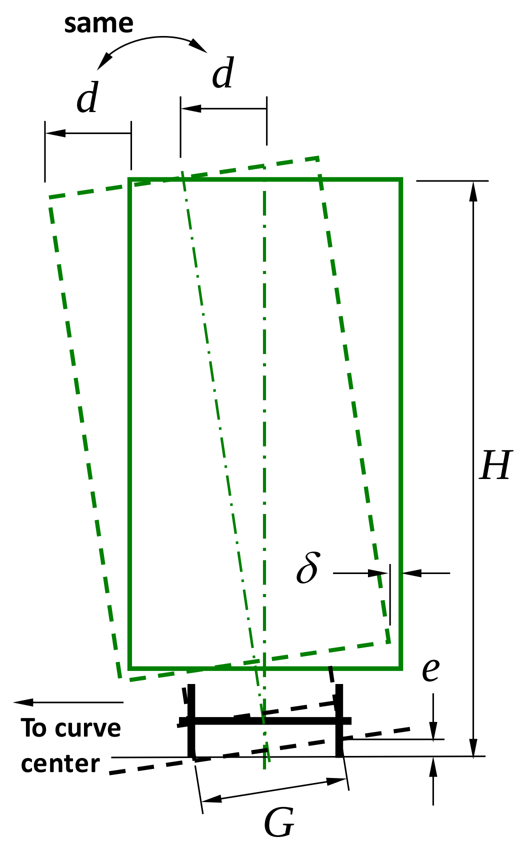

Figure 5: Superelevation Effect on a Curve (end view)

When using equation (2) to make calculations, the superelevation height and track gauge must be expressed in the same units (e.g., inches) to produce the non-dimensional ratio e/G. The distance d will then have the same units as the equipment height units.

Contact Half-Widths

This TN assumes that the following parameters have specified, or given, values:

- Tangent track obstacle clearance
- Tangent track centerline spacing
- Dimensions identified in Figure 3, for two types of equipment
- A specified radius, either for a single track, or the inner of two curved tracks

The AREA equipment definition described earlier makes no mention of the equipment width. Because the spacing values accommodate equipment of all widths, and all equipment fits safely within the tangent spacing values, using actual equipment width is unnecessary. Instead, the equipment half-width is set to the tangent track obstacle clearance or half the tangent track center spacing as needed (more on this shortly). Analysis then proceeds as if each equipment is in contact with its obstacle lines and each is in contact with the other between tracks. Actual contact, of course, does not occur.

NMRA RP-7 and RP-8 recommend specific values for tangent track obstacle and track centerline spacing, respectively. Using these values, the contact half-widths (adjusted equipment half-widths) become:

$C_{obs} = A$ RP-7 dimension **A** (3)

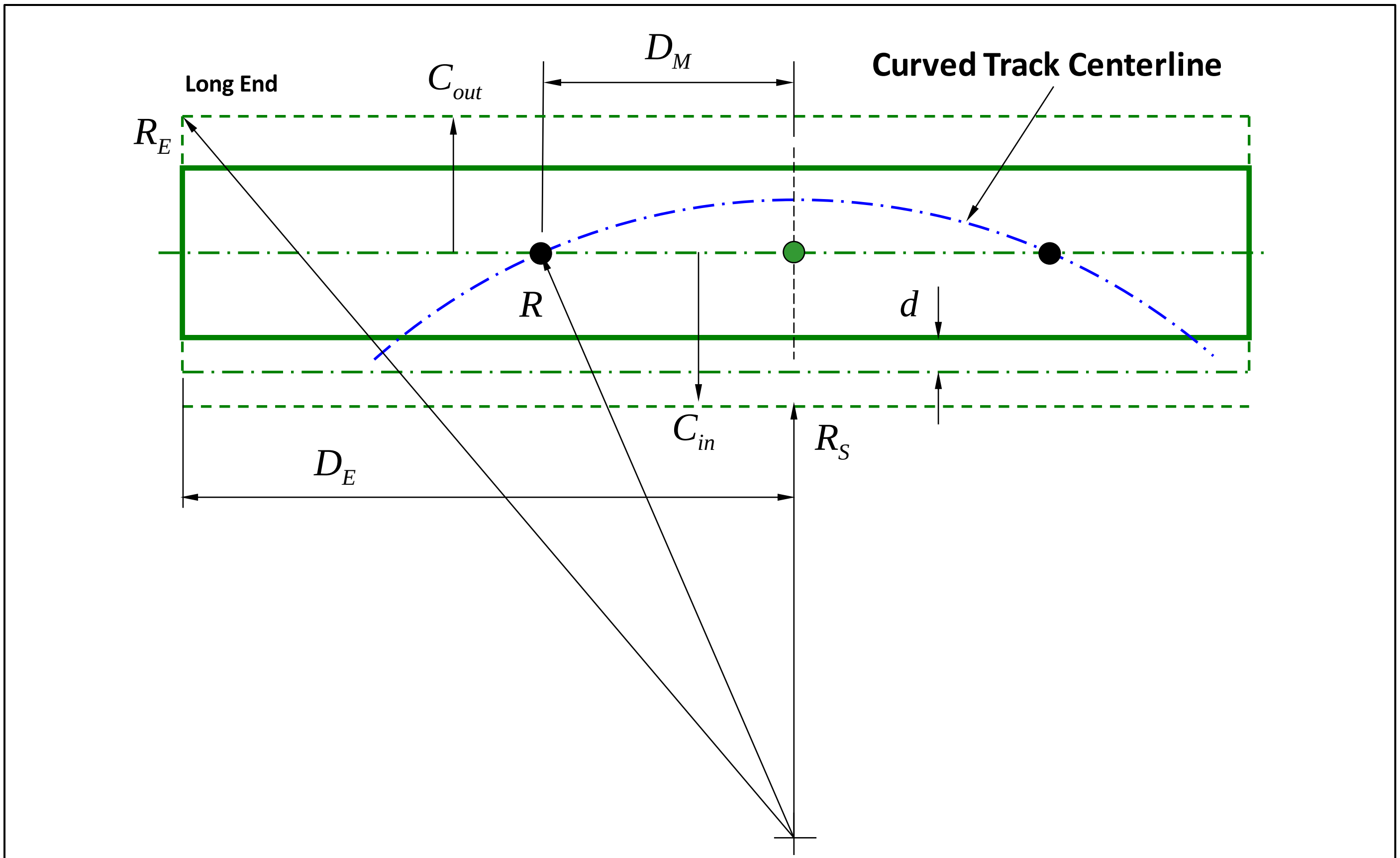

Figure 6: Equipment on Curve (top view)

$$C_{eqp} = \frac{1}{2} S_T \quad \text{half of RP-8 tangent spacing} \qquad (4)$$

The analysis developed here can accept any values as input, but these are logical choices for calculating values for a table expected to appear in RP-8.

Contact Half-Width Usage

Figure 6 illustrates equipment located on a curve. There are two contact half-widths. the first, C_{out}, extends to the outer side obstacle line. The second, C_{in}, extends to the inner side obstacle line. Unlike actual equipment half-widths, they are normally not of equal value. Their assigned values depend on whether the equipment is on a single curve, on the outer of two concentric curves, or on the inner.

For equipment on a curved single track:

$$C_{out} = C_{obs} \qquad (5)$$

$$C_{in} = C_{obs} + d \qquad (6)$$

The additional characters *I* and *O* in the subscripts indicate equipment on the inner and outer tracks, respectively. For equipment on the outer of two tracks:

$$C_{out} = C_{obs} \qquad (7)$$

$$C_{in} = C_{eqp} + d_O \qquad (8)$$

For equipment on the inner of two tracks:

$$C_{out} = C_{eqp} \tag{9}$$

$$C_{in} = C_{obs} + d_I \tag{10}$$

These selections comply with the AREA recommendation *to provide the same clearance*. This means that the equipment on curves will never come closer to a trackside obstacle or to other equipment on concentric curves, than it does on tangents or parallel tangents.

Spacing Equation Development

This section develops equations for the computation of track spacing and obstacle clearance. The first subsection develops the equations for equipment operating on a single curved track.

The equations developed in the second subsection allow for two different types of equipment of known dimensions, one on each of two concentric tracks.

The distance from the pivot mid-point to the long end of the equipment depends on the dimensions from the pivots to their nearest ends, one assumed larger than the other for generality. From the equipment envelope diagram in Figure 1, a covenient expression, to avoid fractions later, is:

$$D_M = \frac{1}{2} D_P \tag{11}$$

If $D_A >= D_B$ then $D_L = D_A$. Otherwise, $D_L = D_B$. As drawn in Figure 1, D_A is larger than D_B, but the opposite is also possible. Once D_L is determined, the long end distance becomes:

$$D_E = D_L + D_M \tag{12}$$

Single Curved Track Obstacle clearance

Figure 6 illustrates a top view of the equipment envelope oriented on a curve, with all radii drawn excessively small for clarity. On a curve, compared to being on a tangent, the outside corner at the long end moves further from the curved track centerline, as does the inside mid-point between the pivot points.

Applying the Pythagorean Theorem to the inner side of the equipment:

$$R^2 = \left(R_S + C_{in}\right)^2 + D_M^2 \tag{13}$$

Substituting equation (6) into equation (13):

$$R^2 = \left(R_S + C_{obs} + d\right)^2 + D_M^2 \tag{14}$$

Solving equation (14) for the radius to the inner side:

$$R_S = \sqrt{R^2 - D_M^2} - C_{obs} - d \tag{15}$$

Notice that R_S, while dependent on the curve radius, superelevation, contact half-width, and pivot point mid-point distance, is independent of the end distance D_E.

Applying the Pythagorean Theorem to the radius to the outer corner of the long end gives:

$$R_E{}^2 = \left(R_S + C_{in} + C_{out}\right)^2 + D_E^2 \qquad (16)$$

Substituting equations (5) and (6) into equation (16):

$$R_E{}^2 = \left(R_S + C_{obs} + d + C_{obs}\right)^2 + D_E^2 \qquad (17)$$

Finally, substitute equation (15) into equation (17) to get:

$$R_E = \sqrt{\left(\sqrt{R^2 - D_M^2} + C_{obs}\right)^2 + D_E^2} \qquad (18)$$

Equation (18) applies to a single curved track *only*. The next section adapts the equations developed here for use on concentric curves.

The curved track obstacle clearance to the outside and to the inside are different and expressed by:

$$S_{SO} = R_E - R \qquad (19)$$

$$S_{SI} = R - R_S \qquad (20)$$

Concentric Curved Track Centerline and Obstacle clearance

Figure 7 illustrates a top view of two types of equipment operating on two concentric curves. At this point the curved track spacing is unknown. The additional subscripts *I* and *O* indicate equipment on the inner and outer tracks, respectively. From Figure 6:

$$R_O = R_I + S_C \qquad (21)$$

Adapting equation (15) for use here:

$$R_{SO} = \sqrt{R_O^2 - D_{MO}^2} - \left(C_{eqp} + d_O\right) \qquad (22)$$

Substitute equation (21) into equation (22):

$$R_{SO} = \sqrt{\left(R_I + S_C\right)^2 - D_{MO}^2} - \left(C_{eqp} + d_O\right) \qquad (23)$$

The radius to the inner side of the equipment on the inner track is:

$$R_{SI} = \sqrt{R_I^2 - D_{MI}^2} - \left(C_{obs} + d_I\right) \qquad (24)$$

Adapting equation (16) for use here:

$$R_{EI}{}^2 = \left(R_{SI} + C_{obs} + d_I + C_{eqp}\right)^2 + D_{EI}^2 \qquad (25)$$

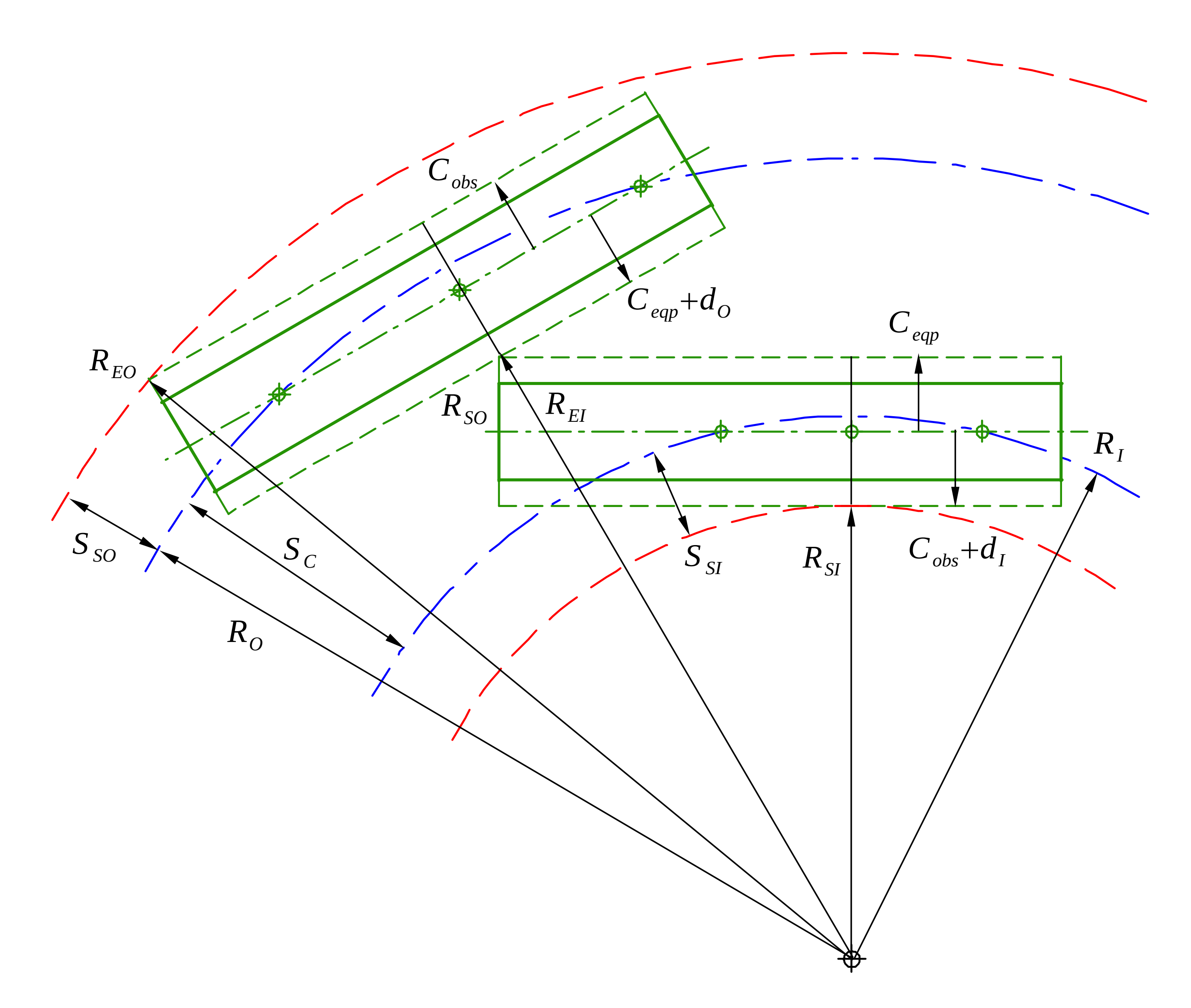

Figure 7: Concentric Curved Track and Obstacle Clearance (top view)

The radius to the outer corner of the equipment on the inner track is then:

$$R_{EI} = \sqrt{\left(R_{SI} + C_{obs} + d_I + C_{eqp}\right)^2 + D_{EI}^2} \qquad (26)$$

Also from Figure 6:

$$R_{SO} = R_{EI} \qquad (27)$$

Substituting equation (23) into equation (27):

$$\sqrt{\left(R_I + S_C\right)^2 - D_{MO}^2} - \left(C_{eqp} + d_O\right) = R_{EI} \qquad (28)$$

Solving for the curved track spacing:

$$S_C = \sqrt{\left[R_{EI} + \left(C_{eqp} + d_O\right)\right]^2 + D_{MO}^2} - R_I \tag{29}$$

Applying the Pythagorean Theorem to find the radius to the outer corner of the equipment on the outer track:

$$R_{EO}^2 = \left(R_{SO} + C_{eqp} + d_O + C_{obs}\right)^2 + D_{EO}^2 \tag{30}$$

Substitute equation (23) into equation (30) and simplify to get:

$$R_{EO} = \sqrt{\left(\sqrt{\left(R_I + S_C\right)^2 - D_{MO}^2} + C_{obs}\right)^2 + D_{EO}^2} \tag{31}$$

The obstacle clearance from the outer track is then:

$$S_{SO} = R_{EO} - R_O \tag{32}$$

The obstacle clearance from the inner track is then:

$$S_{SI} = R_I - R_{SI} \tag{33}$$

Back-substitution in the equations above can eliminate intermediate expressions to produce results in terms of only known parameters. However, this makes the equations unnecessarily complex. For computational purposes, it is simpler to apply the equations in the following sequential order.:

$$R_{SI} = \sqrt{R_I^2 - D_{MI}^2} - \left(C_{obs} + d_I\right) \tag{24}$$

$$R_{EI} = \sqrt{\left(R_{SI} + C_{obs} + d_I + C_{eqp}\right)^2 + D_{EI}^2} \tag{26}$$

$$S_C = \sqrt{\left[R_{EI} + \left(C_{eqp} + d_O\right)\right]^2 + D_{MO}^2} - R_I \tag{29}$$

$$R_{EO} = \sqrt{\left(\sqrt{\left(R_I + S_C\right)^2 - D_{MO}^2} + C_{obs}\right)^2 + D_{EO}^2} \tag{31}$$

$$R_O = R_I + S_C \tag{21}$$

$$S_{SO} = R_{EO} - R_O \tag{32}$$

$$S_{SI} = R_I - R_{SI} \tag{33}$$

Track Arrangement Design Objective

The design objective is to produce a track arrangement that ensures safe operation of all equipment expected to move along a single track, or along two (or more) adjacent tracks. For tangent track, single or parallel, using the specified obstacle clearance and track spacing accomplishes this. Safe operation also means that the contact half-widths for tangent track (or tracks) are the same for curved tracks. The design approach must consider operation on both tangent and curved track arrangements.

Design and Construction Approach

A key phrase in the design objective is *all equipment*. Achieving the design objective first requires examination of the set of all equipment types that must operate on the track arrangement. In that set, there is one type of equipment that has the largest long end distance, and another that has the largest pivot distance. These represent the two equipment extremes that the track arrangement must accommodate.

For identification, these two equipment types carry labels **Equipment 1** and **Equipment 2**, and have known envelope dimensions. These two equipment types comprise the *Design Case* which defines both track spacing and obstacle clearance.

For operation on adjacent tangent tracks, it does not matter which equipment type is on which track. For operation on concentric curved tracks, it does matter which equipment type is on which track. There are two conditions to examine for the Design Case. For **Condition A**, Equipment 1 is on the outer track and Equipment 2 on the inner track. **Condition B** is the opposite, with Equipment 2 on the outer track and Equipment 1 on the inner track. Because which equipment is limiting is not always clear, there are two additional conditions, **Condition C** and **Condition D**, that must be considered for completeness. **Condition C** has Equipment 1 on both tracks, and **Condition D** has Equipment 2 on both tracks. Calculations for each of these four conditions produce *different* curved track centerline spacing and obstacle clearance results.

Examination of the geometry in Figure 7 shows that for an inner curve with a given radius, the curved track spacing is set by placing the equipment with the largest long end distance on the inner track, and the equipment with the largest pivot distance on the outer track. The opposite will produce a smaller curved track spacing, and is thus not limiting. The condition producing the largest curved track spacing is the **Construction** case, i.e., the one that should be built.

Track laying usually occurs before construction or installation of trackside obstacles. Equipment 1 or Equipment 2 may operate on either track. Regardless of which Condition sets the curved track spacing, placing the equipment having the largest pivot distance on the inner track sets the obstacle clearance for the inside of the inner curve. Placing the equipment having the largest long end distance on the outer track sets the obstacle clearance for the outside of the outer curve. This is the opposite of the condition that sets the curved track spacing (previous paragraph). This often causes the **for Construction** outer track obstacle clearance to be slightly smaller than the largest obstacle clearance of any Condition.

When there are more than two concentric curved tracks, numbered sequentially starting with the innermost track, the design approach for tracks 1 and 2 proceeds as described above. The approach is repeated for tracks 2 and 3, using the radius of track 2 as the inner radius for the set of tracks 2 and 3. This continues for the desired number of tracks, and is essentially the process described in [5] that uses the current NMRA S-8 [3]. The process would have to be adapted to reflect any revision, or replacement of, Standard S-8 that may come from consideration of the equations in this TN.

Computer Applications

As mentioned in the introduction, this TN has two companion computer applications, or “apps,” that make track and obstacle clearance calculations using the equations developed above. One is a Visual Basic program (macro) inside an Excel spreadsheet, and the other is a simpler HTML app designed for anyone wishing to determine track and obstacle clearance for a model railroad.

The HTML app simply calculates the track and obstacle clearance given an inner curve radius, equipment geometry, and specified values for the specified tangent and minimum clearances. It is intended for posting on the NMRA website when the appropriate time comes.

The spreadsheet app additionally presents graphs of spacing results as a function of inner curve radius. It also produces tables that may prove useful if some “class-based” presentation in a new or modified Standard or RP becomes desirable.

Model Scale Proportionality Factor	1.0000
ANALYSIS OPTIONS:	
Ouput Units	d
SPECIFICATIONS:	**PROTOTYPE (ft.)**
Tangent Track Center Spacing (ft.)	14.0000
Tangent Track Obstacle Spacing (ft.)	8.0000
TRACK PARAMETERS:	**PROTOTYPE (ft.)**
Inner Curve Radius (in.)	193.1852
Superelevation (ft.)	0.0000
Track Gauge (ft.)	4.7083
ENVELOPE - EQUIPMENT 1:	**PROTOTYPE (ft.)**
Type	AREA 85-ft Pass.
End Distance, DA (ft.)	12.5000
Pivot Distance, DP (ft.)	60.0000
End Distance, DB (ft.)	12.5000
Height, H (ft.)	14.0000
ENVELOPE - EQUIPMENT 2:	**PROTOTYPE (ft.)**
Type	UP 4-8-8-4 Big Boy
End Distance, DA (ft.)	19.2810
Pivot Distance, DP (ft.)	18.2500
End Distance, DB (ft.)	36.2680
Height, H (ft.)	16.2080
GRAPH RADIUS RANGE (for plots):	**PROTOTYPE (ft.)**
Start Radius (ft.)	130.000
Radius Increment (ft.)	2.000
Stop Radius (ft.)	1160.000

Figure 8: Validation Test Input

Computer Application Validation

Figure 8 shows the validation test input values for the spreadsheet app. While the app applies to any modeling scale, the validation test uses prototype dimensions. For the equipment used, it is obvious that **Condition C** and **Condition D** will not be limiting, so they are not shown in what follows.

Figure 9 shows the validation test results the spreadsheet calculated (this is an earlier version of the spreadsheet app which has since been updated. Both versions produce identical results). Normally the spreadsheet displays decimal results to two decimal places (or fractions to the nearest 1/32 inch), but they have been temporarily displayed to six decimal places for the most accurate validation in a CAD program.

A CAD program, here *Design CAD 3D Max*, is used to validate the calculated values by first preparing a drawing like that Figure 7, but drawn to scale using the following steps.

CALCULATED RESULTS:			
ON INNER CURVE RADIUS ONLY:	**for Construction**	**Equipment 1**	**Equipment 2**
Type		AREA 85-ft Pass.	UP 4-8-8-4 Big Boy
Inside Obstacle Spacing	10.343587	10.343587	8.215628
Outside Obstracle Spacing	12.847064	10.147624	12.847064
ON CONCENTRIC CURVED TRACKS:	**for Construction**	**Condition A**	**Condition B**
Equip. on Outer Track		AREA 85-ft Pass.	UP 4-8-8-4 Big Boy
Equip. on Inner Track		UP 4-8-8-4 Big Boy	AREA 85-ft Pass.
Curved Track Spacing	20.983314	20.983314	16.368588
Outer Curve Radius	214.168479	214.168479	209.553754
Inner Track Obstacle Spacing	10.343587	8.215628	10.343587
Outer Track Obstacle Spacing	12.399348	9.954920	12.490643

Figure 9: Validation Test Calculated Results

1. Use the specified inside radius to draw the track centerline of the inner curve.
2. Use the calculated outside radius to draw the track centerline of the outer curve.
3. For each validation case, draw the equipment using the input dimensions on their respective tracks, making sure the outer track equipment is perpendicular to a radial line passing through the outer left corner of the inner track equipment. Locate the long end of the equipment to the left.
4. From the curve center, draw a circular arc that passes through the outer left corner of the outer equipment. This is the obstacle clearance line for the outer track equipment.
5. From the curve center, draw a circular curve that is tangent to the inner dashed line of the inner track equipment. This is the obstacle clearance line for the inner track equipment.

The calculations are valid if the following metrics are all true:

1. The difference in radii of the outer track obstacle line and the outer radius is equal to the calculated outer track obstacle clearance.
2. The difference in the radii of the inner curve radius and the inner track obstacle line is equal to the calculated inner track obstacle clearance.
3. The inner dashed line of the outer equipment and the outer dashed line of the inner equipment intersect at the outer left corner of the inner equipment for **Condition A** and **Condition B** only. **For Construction**, there should be **NO** contact, unless the two types of equipment are identical (here they are not).

To validate the results, the **Condition A**, **Condition B**, and **for Construction** results are laid out in separate CAD drawings using the process described above. Querying the CAD program reveals the obstacle radii and point coordinate values to six decimal places.

Figure 10 illustrates the results for **Condition A**.

CAD outer obstacle line radius	= 224.123399
Calculated outer track radius	= 214.168479
Outer obstacle clearance (diff.)	= 9.954920

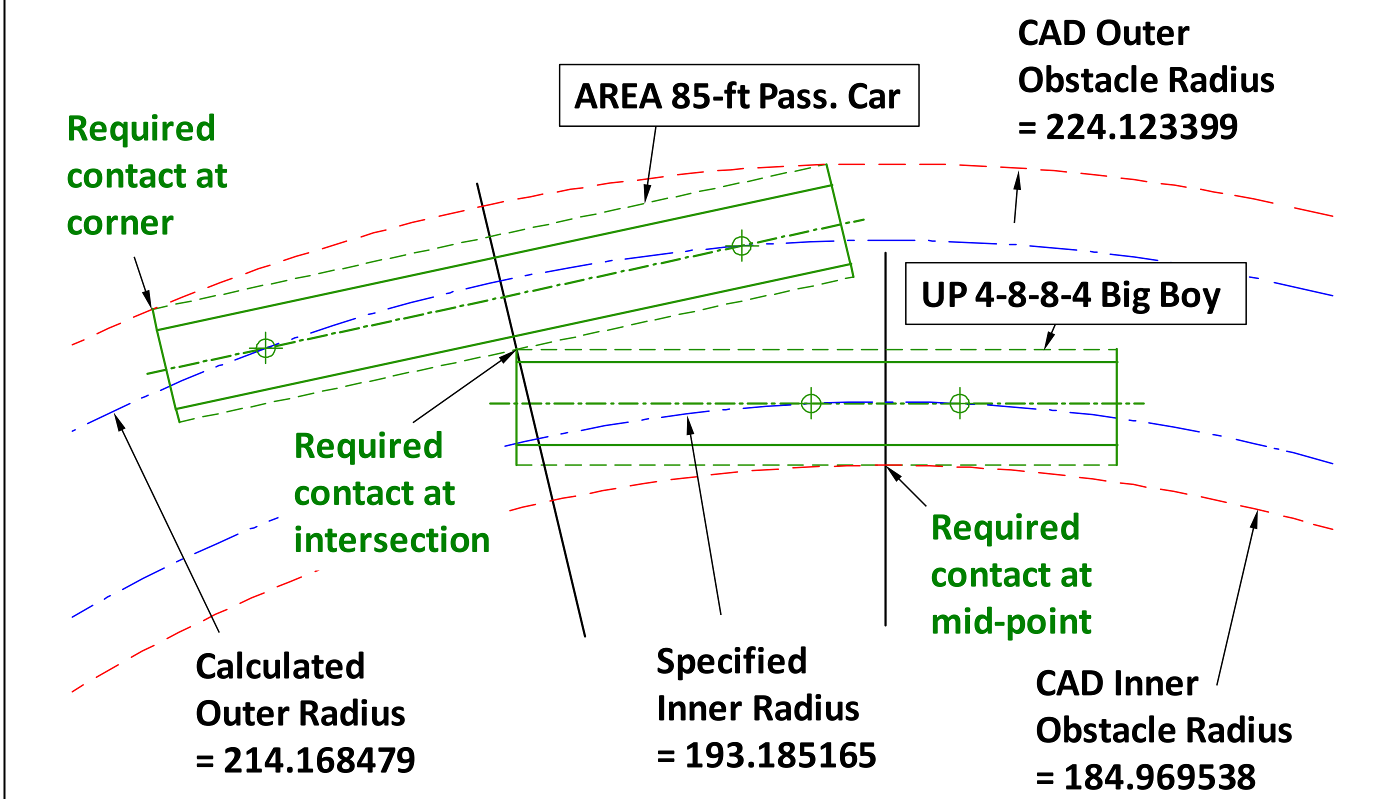

Figure 10: CAD Validation CONDITION A

Calculated obstacle clearance = 9.954920
Spacings match.
Metric 1 satisfied

Specified inner track radius = 193.185165
<u>CAD inner obstacle line radius = 184.969538</u>
Inner obstacle clearance (diff.) = 8.215627
Calculated obstacle clearance = 8.215628
Spacings match, to six significant figures.
Metric 2 satisfied.

	X	Y
Mid-pivot coordinates	-45.393000	-45.393000
Corner coordinates	199.969538	199.969538

Lines intersect because coordinates match.
Metric 3 satisfied.

All metrics satisfied, so **CONDITION A calculations are valid.**

Figure 11 illustrates the results for **Condition B**.

CAD outer obstacle line radius = 222.044397
<u>Calculated outer track radius = 209.553754</u>

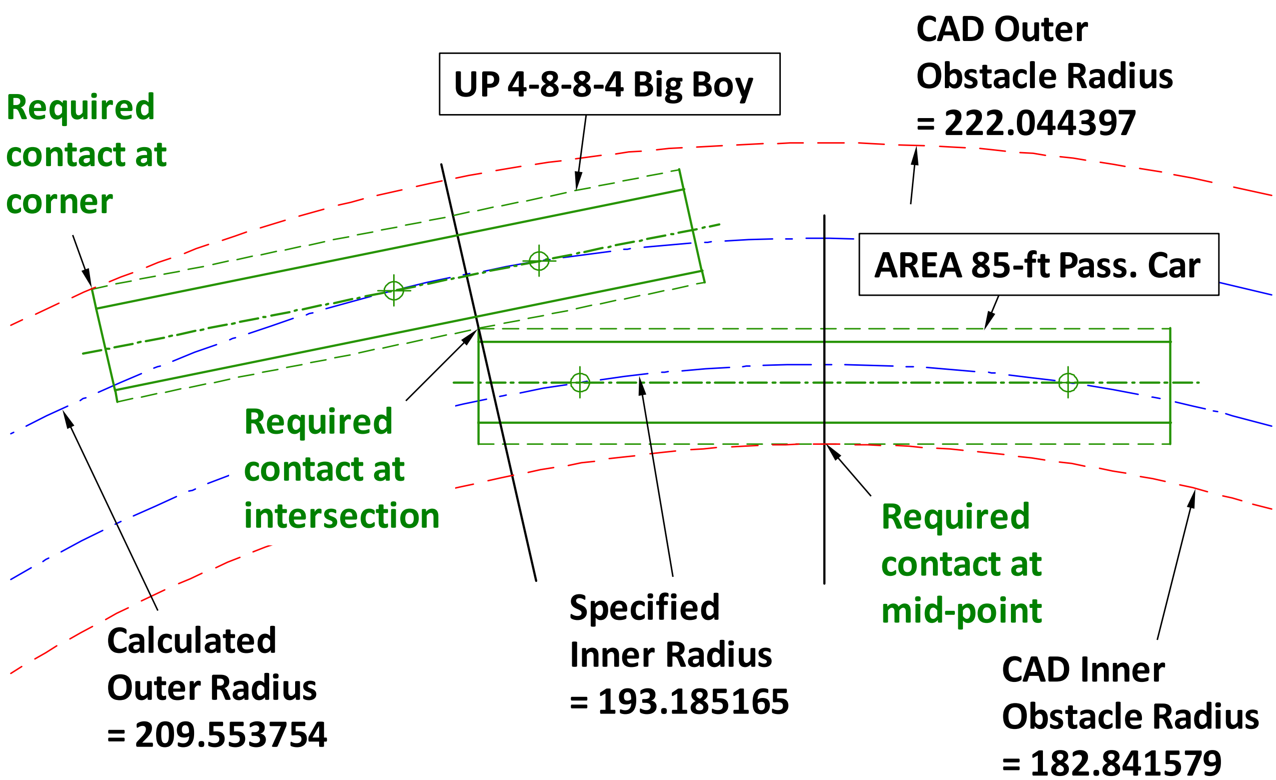

Figure 11: CAD Validation CONDITION B

Outer obstacle clearance (diff.) = 12.490643
Calculated obstacle clearance = 12.490643
Spacings match.
Metric 1 satisfied

Specified inner track radius = 193.185165
<u>CAD inner obstacle line radius = 184.969538</u>
Inner obstacle clearance (diff.) = 8.215627
Calculated obstacle clearance = 8.215628
Spacings match, to six significant figures.
Metric 2 satisfied.

	X	**Y**
Mid-pivot coordinates	-45.393000	-45.393000
Corner coordinates	199.969538	199.969538

Lines intersect because coordinates match.
Metric 3 satisfied.

All metrics satisfied, so **CONDITION B calculations are valid.**

Figure 12 show the results **for Construction**.

CAD outer obstacle line radius = 226.567827

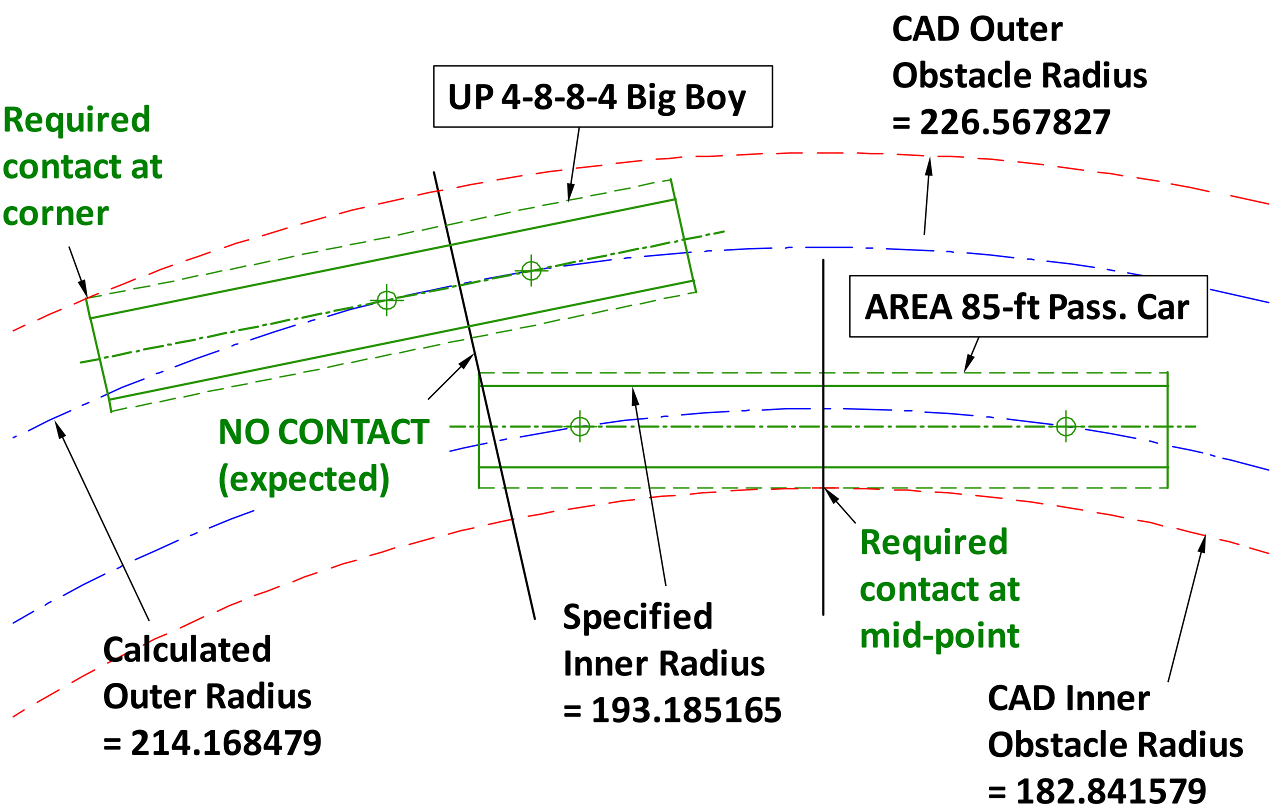

Figure 12: CAD Validation for CONSTRUCTION

Calculated outer track radius = 214.168479
Outer obstacle clearance (diff.) = 12.399348
Calculated obstacle clearance = 12.399348
Spacings match.
Metric 1 satisfied

Specified inner track radius = 193.185165
CAD inner obstacle line radius = 182.841579
Inner obstacle clearance (diff.) = 10.343586
Calculated obstacle clearance = 10.343587
Spacings match, to seven significant figures.
Metric 2 satisfied.

	X	Y
Mid-pivot coordinates	-43.470117	-42.500000
Corner coordinates	202.357567	197.841579

Lines DO NOT intersect because coordinates DO NOT match, AS EXPECTED.
Metric 3 satisfied.

All metrics satisfied, so **for CONSTRUCTION calculations are valid.**

Because all metrics for all three conditions are valid, the spreadsheet calculations are valid.

TRACK CENTER AND OBSTACLE SPACING CALCULATOR

by: Van S. Fehr, DRAFT 02, March 2017

Instructions:

1. First enter the **SPECIFICATIONS**, **TRACK PARAMETERS** and **EQUIPMENT ENVELOPE** information in the yellow boxes.
2. If your input has units of inches, enter an **F** in the **Output Units Format** box to display results in whole and fractional inches (e.g., 2 3/32). Otherwise, enter a **D** to display results in decimal format for any input units, including inches. You may use an upper OR lower case letter.
3. When complete, select, click, or tap the orange **CALCULATE SPACING** button to compute the calculated track center and obstacle spacing dimensions, and display them in the light blue boxes. Use the dimensions displayed in the light green boxes for actual construction.

See NMRA TBD *Track Center and Obstacle Spacing* for detailed explanations and instructions.

SPECIFICATIONS:		**TRACK PARAMETERS:**	
Tangent Spacing	14.0	**Inner Curve Radius**	193.1852
Obstacle Spacing	8.0	**Superelevation**	0.00
Output Units Format	D	**Track Gauge**	4.708333

EQUIPMENT ENVELOPES:	**Equipment 1**	**Equipment 2**
Type	AREA 85-ft Passenger	UP 4-8-8-4 Big Boy
End Distance A	12.50	19.281
Pivot Distance	60.00	18.250
End Distance B	12.50	36.268
Height	14.000	16.208

CALCULATE SPACING

ON INNER CURVE RADIUS ONLY:	**For Construction**	**Equipment 1**	**Equipment 2**
Type		AREA 85-ft Passenger	UP 4-8-8-4 Big Boy
Inside Obstacle Spacing	10.343586	10.343586	8.215628
Outside Obstacle Spacing	12.847063	10.147624	12.847063

ON CONCENTRIC CURVED TRACKS:	**For Construction**	**Condition A**	**Condition B**
Equipment on Outer Track		AREA 85-ft Passenger	UP 4-8-8-4 Big Boy
Equipment on Inner Track		UP 4-8-8-4 Big Boy	AREA 85-ft Passenger
Curved Track Center Spacing	20.983312	20.983312	16.368588
Outer Curve Radius	214.168512	214.168512	209.553788
Inner Track Obstacle Spacing	10.343586	8.215628	10.343586
Outer Track Obstacle Spacing	12.399347	9.954920	12.490643

Figure 13: HTML App Validation

Finally, Figure 13 shows the validation test results for the HTML app, using the same input values used in the spreadsheet app (This is also an earlier version of the HTML app. Both versions produce identical results). The HTML app results match those of the spreadsheet app shown in Figure 9 to six significant figures, validating the HTML calculations. The HTML app normally displays results to two decimal places, but six are again displayed here for validation testing.

Observations

Examination of the equations and the validation results lead to the following observations:

1. Curved track and obstacle clearance depends on the specified tangent track and tangent obstacle clearance, the equipment geometry, the amount of superelevation, and the inner curve radius.
2. For equipment operating on a curve of any given radius, the larger the pivot distance, the further the inner side of the equipment extends inward from the curve centerline.
3. For equipment operating a curve of any given radius, the larger the long-end distance, the further the outer corner of the long end extends outward from the curve centerline.
4. When the equipment operating on two adjacent curves are *different* types, the curved track spacing is set by having the equipment with the largest long-end distance on the inner track, and the equipment with the largest pivot distance on the outer track. The opposite will produce a smaller curved track spacing, and is thus not limiting.
5. Once curved track spacing is set per observation 4 immediately above, the inner and outer curve obstacle clearance is set when the equipment is on tracks opposite those in observation 4.
6. Placing equipment of the *same* type on each curve produces a smaller curved track spacing than equipment of different types, and is thus not limiting.

References

1. (No named contributors) “Interurban Clearance and Track Centers”, NMRA Standard S-6, July 1986
2. R. Gaines, B. Barnt, “Clearances”, NMRA Standard S-7, Feb 2012
3. (No named contributors), “Track Centers”, NMRA Standard S-8, Jul 2002
4. William W. Hay, *Railroad Engineering*, 2nd ed., John Wiley and Sons, Inc, 1982
5. Van S. Fehr, “How to plan concentric curve with easements,” *Model Railroad Planning 2011*, pp. 62-65, Kalmbach Publishing

Change Record

Jan 2017	DRAFT 01 (Van S. Fehr)
Mar 2017	DRAFT 02 (Van S. Fehr)
May 2017	DRAFT 03 (Van S. Fehr)
Jul 2017	NMRA BOD Approved

APPENDIX

Table 1: Mathematical Variables

Variable	Definition	Variable	Definition
A	Dimension **A** from NMRA RP-7. Also, the minimum obstacle clearance	d_O	The inward tilt due to superelevation at the top of equipment envelope for equipment on outer track
C_{eqp}	Equipment clearance, taken as one-half S_T	δ	The negligible inward tilt at the bottom of equipment envelope due to superelevation
C_{in}	Inner half-width	e	Superelevation height of outer rail
C_{obs}	Trackside obstacle clearance	G	Track gauge
C_{out}	Outer half-width	H	Equipment height above railheads
D_A	Distance from one end to nearest pivot point	R	Curved track centerline radius
D_B	Distance from the other end to nearest pivot point	R_E	Radius to the outer corner of the long end
D_E	Distance from the pivot mid-point to the long end	R_{EI}	Radius to the outer corner of the long end for equipment on inner curve
D_{EI}	Distance from the pivot mid-point to the long end for equipment on inner track	R_I	Inner curve radius
D_{EO}	Distance from the pivot mid-point to the long end for equipment on outer track	R_O	Outer curve radius
D_L	The larger of the distances D_A and D_B	R_S	Radius to equipment side on inside of curve, and at the pivot mid-point
D_M	One-half the distance between pivot points	R_{SO}	Radius to equipment side on inside of curve, and at the pivot mid-point, for equipment on outer curve
D_{MI}	One-half the distance between pivot points for equipment on inner track	S_C	Calculated curved track centerline spacing
D_{MO}	One-half the distance between pivot points for equipment on outer track	S_S	Minimum obstacle clearance. Also, dimension **A** from NMRA RP-7.
D_P	Distance between pivot points.	S_{SO}	Trackside obstacle clearance for outer corner of equipment long-end on outer track
d	The inward displacement at the top of equipment envelope due to superelevation	S_{SI}	Trackside obstacle clearance for equipment inner side on inner track
d_I	The inward tilt due to superelevation at the top of equipment envelope for equipment on inner track	S_T	Specified minimum tangent track spacing from NMRA RP-8